MATH 135 — QUIZ 9 SOLUTIONS — JAMES HOLLAND
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Question 1. Using l'Hôpital's rule, evaluate $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}}$.

Solution .:.

Since $\lim_{x\to\infty} \ln(x) = \infty$ and $\lim_{x\to\infty} \sqrt{x} = \infty$, we can apply l'Hôpital's rule and conclude

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} = \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})}.$$

To evaluate this limit, note that

$$\frac{1/x}{1/(2\sqrt{x})} = \frac{2\sqrt{x}}{x} = \frac{2}{\sqrt{x}}.$$

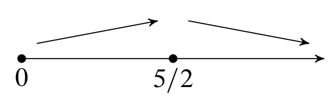
Hence taking the limit as x goes to ∞ yields 0:

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt{x}} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0.$$

Question 2. Find the maximum area of a rectangle with perimeter 10.

Solution .:.

A rectangle with side lengths $x > 0$ and $y > 0$ has perimeter $P(x, y) = x + x + y + y = 2x + 2y$ and area $A(x, y) = x \cdot y$. If $P(x, y) = 10$, then $2x + 2y = 10$ so that $y = 5 - x$. Therefore $A(x) = x \cdot (5 - x) = 5x - x^2$. To minimize this, $A'(x) = 5 - 2x$ which is 0 iff $x = 5/2$. This is a maximum for A by the first derivative test: for $x < 5/2$ (e.g. $x = 1$) $A'(x) > 0$ while $A'(x) < 0$ for $x > 5/2$ (e.g. $x = 3$). This is represented below:



Hence $x = 5/2$ yields a maximum area of $A(5/2) = (5/2)^2 = 25/4$.